


SOALAN RAMALAN

MATEMATIK TAMBAHAN

KERTAS 1

FUNGSI TRIGONOMETRI
TRIGONOMETRY FUNCTION

SIR VEN
(GURU ADIWIRA KEBANGSAAN 2019)

SPM 2023

The equation $2 \tan^2 \mu + \sec^2 \mu = 5 - 4 \tan \mu$,
Persamaan $2 \tan^2 \mu + \sec^2 \mu = 5 - 4 \tan \mu$,

where/ *di mana* $0° \leqslant \mu \leqslant 90°$.

(a) Show that/ *Tunjukkan bahawa*

$$3 \tan^2 \mu + 4 \tan \mu - 4 = 0$$

(b) Hence, find the value of $\tan \mu$.
Seterusnya, cari nilai $\tan \mu$.

FUNGSI TRIGONOMETRI
TRIGONOMETRY FUNCTION

Solve the equation $2\sec^2 x + 9\tan x - 7 = 0$ for $0° \leq x \leq 360°$.

Selesaikan persamaan $2\,\text{sek}^2 x + 9\tan x - 7 = 0$ *untuk* $0° \leq x \leq 360°$.

) Solve the equation $2 \sin x = \tan x$ for $0° \leqslant x \leqslant 180°$.
Selesaikan persamaan $2 \sin x = \tan x$ untuk $0° \leqslant x \leqslant 180°$.

It is given that $\tan \theta = t$ where t is a constant and $0° \leqslant \theta \leqslant 90°$.

Diberi bahawa $\tan \theta = t$ *dengan keadaan* t *ialah pemalar dan* $0° \leqslant \theta \leqslant 90°$.

Express in terms of t,

Ungkapkan dalam sebutan t,

(a) $\cos (180° - \theta)$

kos $(180° - \theta)$

(b) $\text{cosec } 2\theta$

kosek 2θ

It is given that $\sin A = -\frac{4}{5}$, where $90° < A < 270°$. Find

Diberi bahawa $\sin A = -\frac{4}{5}$, *dengan keadaan* $90° < A < 270°$. *Cari*

(a) cosec A.
kosek A.

(b) $\sin 2A$.

Solve the equation $10 \cos \theta = 7 - \sec \theta$ for $0° \leqslant \theta \leqslant 180°$.
Selesaikan persamaan 10 kos $\theta = 7 -$ sek θ *untuk* $0° \leqslant \theta \leqslant 180°$.

Given $\sin^2 x = m$ and $\cos^2 x = n$, express cosec x in terms of m and n.
Diberi $\sin^2 x = m$ *dan* $\text{kos}^2 x = n$, *ungkapkan* kosek x *dalam sebutan* m *dan* n.

Given $\sin x = y$, where y is a constant and $0° \leqslant x \leqslant 90°$. Find in terms of y,
Diberi $\sin x = y$, *dengan keadaan* y *ialah pemalar dan* $0° \leqslant x \leqslant 90°$. *Cari dalam sebutan* y,

(a) $\cot^2 x$.
$\text{kot}^2 x$.

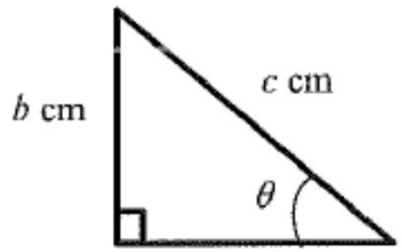
(b) $\cos (180° + x)$.
$\text{kos} (180° + x)$.

FUNGSI TRIGONOMETRI
TRIGONOMETRY FUNCTION

(a) Buktikan bahawa $\frac{1-\text{kos } 2x}{1+\text{kos } 2x} = \tan^2 x$. [3 *markah*]

Prove that $\frac{1-\cos 2x}{1+\cos 2x} = \tan^2 x$. [3 *marks*]

(b) Dengan menggunakan Rajah 2 , tunjukkan bahawa $sin^2\theta + kos^2\theta = 1$.
By using Diagram 2, *show that* $sin^2\theta + kos^2\theta = 1$.



Rajah 2 / *Diagram* 2

[3 *markah*]
[3 *marks*]

Diberi bahawa $\tan A = k$ dengan keadaan $0° < A < \frac{\pi}{2}$, ungkapkan $\tan\left(\frac{\pi}{2} - A\right)$ dalam sebutan k. [2 *markah*]

Given that $\tan A = k$ *for* $0° < A < \frac{\pi}{2}$, *express* $\tan\left(\frac{\pi}{2} - A\right)$ *in terms of* k. [2 *marks*]

(a) Selesaikan persamaan $2\kos^2 2x + 3\sin 2x - 3 = 0$ untuk $0 \le x \le 360^\circ$.

Solve the equation $2\kos^2 2x + 3\sin 2x - 3 = 0$ *for* $0 \le x \le 360^\circ$.

[4 markah
[4 marks

(b) Diberi bahawa $\tan 2x = \frac{5}{12}$ dengan keadaan x adalah sudut cakah. Cari nilai $\kos^2 x$.

Given $\tan 2x = \frac{5}{12}$ *such that* x *is an obtuse angle. Find the value of* $\cos^2 x$.

[3 markah
[3 marks

Lakarkan graf bagi $y = 3 \text{ kos } 2x$ bagi $0 \leq x \leq \pi$. Seterusnya, tentukan nilai k supaya kos $2x = 2k$ mempunyai satu penyelesaian sahaja bagi $0 \leq x \leq \pi$. [5 *markah*]

Sketch the graph of $y = 3 \text{ kos } 2x$ *for* $0 \leq x \leq \pi$. *Hence, find the value of* k *so that* $\cos 2x = 2k$ *has only one solution for* $0 \leq x \leq \pi$. [5 *marks*]

FUNGSI TRIGONOMETRI
TRIGONOMETRY FUNCTION

(a) Lakar graf $y = -2 - |3\sin 2x|$ bagi $0 \le x \le 2\pi$.

Sketch the graph $y = -2 - |3\sin 2x|$ *for* $0 \le x \le 2\pi$.

(b) Selesaikan $\text{kos}\, x \sin x = \dfrac{\sqrt{3}}{6}$ untuk $0 \le x \le 360$.

Solve $\cos x \sin x = \dfrac{\sqrt{3}}{6}$ *for* $0 \le x \le 360$.

(a) $2\tan^2\mu + \sec^2\mu = 5 - 4\tan\mu$

$2\tan^2\mu + 1 + \tan^2\mu + 4\tan\mu - 5 = 0$

$3\tan^2\mu + 4\tan\mu - 4 = 0$ (Shown)

(b) $3\tan^2\mu + 4\tan\mu - 4 = 0$

$(3\tan^2\mu - 2)(\tan\mu + 2) = 0$

$3\tan^2\mu - 2 = 0$ or $\tan^2\mu + 2 = 0$

$\tan\mu = \frac{2}{3}$ $\tan\mu = -2$

$\mu = 33.69°$ (Rejected)

15. $2\sec^2 x + 9\tan x - 7 = 0$

$2(1 + \tan^2 x) + 9\tan x - 7 = 0$

$2 + 2\tan^2 x + 9\tan x - 7 = 0$

$2\tan^2 x + 9\tan x - 5 = 0$

$(2\tan x - 1)(\tan x + 5) = 0$

$\tan x = 2$ or $\tan x = -5$

$x = 26.57°,\ 206.57°$ $x = 101.31°,\ 281.31°$

$\therefore x = 26.57°, 101.31°, 206.57°, 281.31°$

$2\sin x = \tan x$

$2\sin x = \frac{\sin x}{\cos x}$

$2\sin x\cos x = \sin x$

$\sin x(2\cos x - 1) = 0$

$\sin x = 0$ or $2\cos x - 1 = 0$

$x = 0°, 180°$ or $\cos x = \frac{1}{2}$

$x = 60°$

$\tan\theta = t$

$\sin\theta = \frac{t}{\sqrt{1+t^2}}$

$\cos\theta = \frac{1}{\sqrt{1+t^2}}$

(a) $\cos(180° - \theta) = \cos 180°\cos\theta + \sin 180°\sin\theta$

$= (-1)\left(\frac{1}{\sqrt{1+t^2}}\right) + 0$

$= -\frac{1}{\sqrt{1+t^2}}$

(b) $\operatorname{cosec} 2\theta = \frac{1}{\sin 2\theta}$

$= \frac{1}{2\sin\theta\cos\theta}$

$= \frac{1}{2\left(\frac{t}{\sqrt{1+t^2}}\right)\left(\frac{1}{\sqrt{1+t^2}}\right)}$

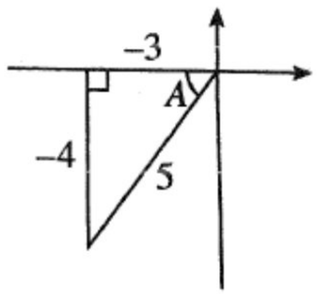
$= \frac{1+t^2}{2t}$

(a) $\text{cosec } A = -\frac{5}{4}$

(b) $\sin 2A = 2 \sin A \cos A$

$= 2\left(-\frac{4}{5}\right)\left(-\frac{3}{5}\right)$

$= \frac{24}{25}$



3 $10 \cos \theta = 7 - \sec \theta$

$10 \cos \theta = 7 - \frac{1}{\cos \theta}$

$10 \cos^2 \theta = 7 \cos \theta - 1$

$10 \cos^2 \theta - 7 \cos \theta + 1 = 0$

$(2 \cos \theta - 1)(5 \cos \theta - 1) = 0$

$\cos \theta = \frac{1}{2}$ or $\cos \theta = \frac{1}{5}$

$\theta = 60°$ $\theta = 78.46°$

Hence, $\theta = 60°$ and $78.46°$.

$\text{cosec}^2 x = 1 + \cot^2 x$

$= 1 + \frac{\cos^2 x}{\sin^2 x}$

$= 1 - \frac{n}{m}$

$\text{cosec } x = \sqrt{1 - \frac{n}{m}}$

(a) $\cot^2 x = \text{cosec}^2 x - 1$

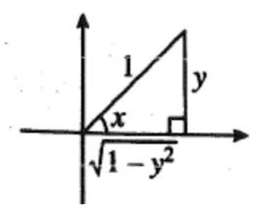
$= \frac{1}{\sin^2 x} - 1$

$= \frac{1}{y^2} - 1$

(b) $\cos (180° + x) = \cos 180° \cos x - \sin 180° \sin x$

$= (-1)(\sqrt{1 - y^2}) - (0)(y)$

$= -\sqrt{1 - y^2}$



7 (a)

$$= \frac{1-\left(1-2\sin^2 x\right)}{1+\left(2\text{kos}^2 x-1\right)}$$

$$= \frac{2\sin^2 x}{2\text{kos}^2 x}$$

$$= \tan x$$

(b)

$a^2+b^2=c^2$

$\frac{a^2}{c^2}+\frac{b^2}{c^2}=\frac{c^2}{c^2}$

$\left(\frac{a}{c}\right)^2+\left(\frac{b}{c}\right)^2=1$

$(kos\ \theta)^2+(\sin\theta)^2=1$

$\sin^2\theta+kos^2\theta=1$

$$\tan\left(\frac{\pi}{2}-A\right)=\text{kot}\,A$$

$$=\frac{1}{k}$$

$2kos^2 2x + 3sin\ 2x - 3 = 0$

$2(1 - sin^2 2x) + 3sin\ 2x - 3 = 0$

$-2sin^2 2x + 3sin\ 2x - 1 = 0$

$2sin^2 2x - 3sin\ 2x + 1 = 0$

Let $sin\ 2x = y$

$2y^2 - 3y + 1 = 0$

$(2y - 1)(y - 1) = 0$

$y = 1$ or $y = \frac{1}{2}$

$sin\ 2x = 1$ $\quad sin\ 2x = \frac{1}{2}$

$2x = 30°, 150°, 390°, 510°$ $\quad 2x = 90°, 450°$

$x = 15°, 75°, 195°, 255°$ $\quad x = 45°, 225°$

Hence, $x = 15°, 75°, 195°, 255°, 45°, 225°$

$\tan 2x = \frac{5}{12}$

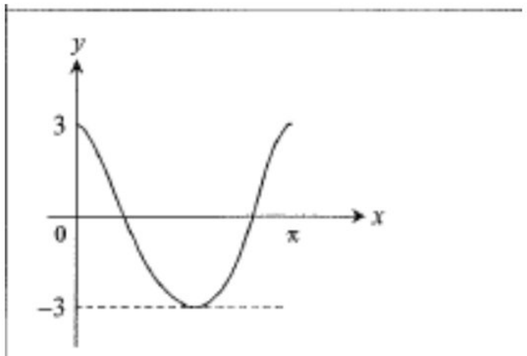
$\text{kos}\,2x = -\frac{12}{13}$

$\text{kos}\,2x = 2\text{kos}^2 x - 1$

$-\frac{12}{13} = 2\text{kos}^2 x - 1$

$2\text{kos}^2 x = \frac{\left(-\frac{12}{13}\right)+1}{2}$

$\text{kos}^2 x = \frac{1}{26}$



kos $2x = 2k$

3 kos $2x = 6k$

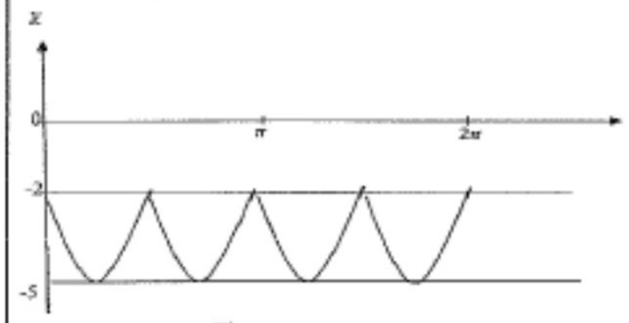
$y = 6k$ mempunyai satu penyelesaian di $y=-3$

$6k = -3$

$k = -\frac{1}{2}$

FUNGSI TRIGONOMETRI
TRIGONOMETRY FUNCTION

Bentuk, amplitud, kala



$$2\text{kos } x \sin x = \frac{\sqrt{3}}{3}$$

$$\sin 2x = \frac{\sqrt{3}}{3}$$

$$2x = 35.26^0, 144.74^0, 395.26^0, 504.74^0$$

$$x = 17.63^0, 72.37^0, 197.63^0, 252.37^0$$